mannian hypothesis that contradicts it must be false. This follows inevitably by the logical law of *Excluded Middle*, according to which if one of two propositions that mutually contradict each other is true, the other must be false.

According to the Euclidian view the longer a straight line is the further apart are its ends.

According to the Riemannian view a straight line may be lengthened until its ends approach and ultimately meet.

The hypothesis of Riemann and the 2nd postulate of Euclid contradict each other. Hence, both cannot be true. To accept both is to discredit logical law. To say that we do not know which is true is to confess that we are not in possession of geometrical Science.

According to the laws of logical deduction, if Euclid's postulate 2 is false, the geometrical System derived from it is not true.

On the other hand, if the assumption that contradicts Euclid's postulate 2 is false, the system logically deduced from it is not true. Sound geometrical propositions are not obtained by logical deduction from false data.

According to the Riemannian hypothesis the angle sum of a rectilineal triangle is greater than two right angles. But Lobatschewsky proves in his theorem 19 that the angle sum of a rectilineal triangle cannot be greater than two right angles. The hypotheses of Lobatschewsky and Riemann, therefore, are seen to clash with each other as well as with the axioms, postulates and theorems of Euclid's Elements.

The chords of arcs of circles are not identical with the arcs subtended by them. Hence *rectilineal* triangles should not be treated as identical with *spherical* triangles. This statement holds whatever the length of the radius of the sphere may be. The radius of every sphere has *two* ends, one at the centre and the other at the surface. But every straight line with *two* ends is *finite.* We are now face to face with Postulate III. of Euclid's Elements.

SUBSTITUTION GROUPS.

THE CONSTRUCTION OF INTRANSITIVE GROUPS CONTINUED.

Before seeking all of the possible intransitive groups of degree* six it seems well to call attention to several facts which may be employed to advantage in this work. To illustrate we shall employ a group which was given before, viz.

* The degree of a group is equal to the number of letters it involves. Thus $(abcd)$ pos is of the fourth degree.

$$
(abcd)\text{pos.}\dagger = 1 \quad \begin{array}{lll} & abc & acb \\ ab.cd & bdc & bcd \\ ac.bd & adb & abd \\ ad.bc & acd & adc. \end{array}
$$

If t and s represent any two substitutions then is

$$t^{-1}st$$

(where t^{-1} represents the substitution which reverses the operation indicated by t) called the *conjugate of s with respect to t*. t and t^{-1} are said to be the *inverse* of each other, since $tt^{-1}=t^{-1}t=1$.

We proceed to find the conjugates of $ab.cd$ with respect to the other substitutions of $(abcd)$ pos. We obtain the following results:

$$
\begin{array}{lll} ac.bd & ab.cd & ac.bd=ab.cd \\ abc & ab.cd & acb=ac.bd \\ acb & ab.cd & abc=ad.bc\ddagger \end{array}
$$

It can readily be verifid that all the substitutions in one of the above columns transform $ab.cd$ into the same substitution.

Definition. If all the substitutions of a group transform all the substitutions of a subgroup into substitutions of the subgroup, the subgroup is called a self conjugate subgroup of the given group.

$$1,\quad ab.cd,\quad ac.bd,\quad ad.bc$$

constitute a self conjugate subgroup of $(abcd)$ pos, while the subgroups

$$1,\ ab.cd \ \text{ and } \ 1,\ abc,\ acb$$

are not self conjugate. If we exclude identity and the entire group from the subgroups, it can easily be verified that only one of the eight subgroups of the given group is self conjugate.

Since all the intransitive groups of a given degree n can be obtained by combining transitive groups such that the sum of their degrees is equal to n it follows that all the intransitive groups of degree six can be found by combining

(1) a transitive group of degree three with a transitive group of degree three,

(2) a transitive group of dregree two with a transitive group of degree four, and

(3) a transitive group of degree two with two transitive groups of degree two.*

We proceed to find the intransitive groups for each of these divisions separately. We shall thus not only find all the groups but also each group only once since it is evidently impossible for one group to belong to two of these divisions.

[To be continued.]

†By $(abc....l)$ all we mean all the substitutions that can be formed with the letters $a, b, c, ..., l$ and by $(abc....l)$ pos we mean the subgroup of the preceding group which involves only its positive substitutions; i. e. all its substitutions which indicate an even number of interchanges of two letters. In place of (abc) pos it is customary to write (abc) cyc or merely (abc).

‡The conjugate may be obtained by replacing each letter by the letter which follows it in the substitution with respect to which it is conjugate. For let $s=a_1\, a_2\, a_3 ....$ and let $b_1\, b_2\, b_3\, ...$ be the letters in order which in t succeed the given letters of s. Then $t^{-1}st$ replaces b_1 by a_1, a_1 by a_2, and a_2 by b_2: i. e. it replaces b_1 by b_2 and similarly it replaces b_2 by b_3. If a_α is not explicitedly found in t we have to observe that $b_\alpha=a_\alpha$ in using this method.

*Every group is transitive whose degree (n) satisfies the inequality $n<4$.